CLASA A X-A

SUBIECTUL 1.

Dacă $a,b,c > 1$, să se demonstreze inegalitatea:

$$a^{\sqrt[3]{\log_a b}+\sqrt[3]{\log_a c}}+b^{\sqrt[3]{\log_b a}+\sqrt[3]{\log_b c}}+c^{\sqrt[3]{\log_c a}+\sqrt[3]{\log_c b}}\leq\frac{(a+b+c)^2}{3}$$

Soluție:

Deoarece $a,b,c > 1$ atunci $\log_a b>0$, $\log_a c>0$, $\log_b a>0$, $\log_b c>0$, $\log_c a>0$, $\log_c b>0$ **(1p)**

Folosind inegalitatea mediilor obținem:

$$\sqrt[3]{\log_a b}+\sqrt[3]{\log_a c}=\sqrt[3]{1\cdot 1\cdot \log_a b}+\sqrt[3]{1\cdot 1\cdot \log_a c}\leq\frac{1+1+\log_a b}{3}+\frac{1+1+\log_a c}{3} \quad \textbf{(2p)}$$

$$\frac{1+1+\log_a b}{3}+\frac{1+1+\log_a c}{3}=\frac{\log_a a^2b}{3}+\frac{\log_a a^2c}{3}$$

$$\frac{\log_a a^2b}{3}+\frac{\log_a a^2c}{3}=\frac{\log_a a^4bc}{3}=\log_a a\sqrt[3]{abc} \quad \textbf{(1p)}$$

Cum $a,b,c > 1$ avem:

$$a^{\sqrt[3]{\log_a b}+\sqrt[3]{\log_a c}}+b^{\sqrt[3]{\log_b a}+\sqrt[3]{\log_b c}}+c^{\sqrt[3]{\log_c a}+\sqrt[3]{\log_c b}}\leq$$

$$\leq a^{\log_a a\sqrt[3]{abc}}+b^{\log_b b\sqrt[3]{abc}}+c^{\log_c c\sqrt[3]{abc}} \quad \textbf{(1p)}$$

$$a^{\log_a a\sqrt[3]{abc}}+b^{\log_b b\sqrt[3]{abc}}+c^{\log_c c\sqrt[3]{abc}}=a\sqrt[3]{abc}+b\sqrt[3]{abc}+c\sqrt[3]{abc} \quad \textbf{(1p)}$$

$$a\sqrt[3]{abc}+b\sqrt[3]{abc}+c\sqrt[3]{abc}=(a+b+c)\sqrt[3]{abc}\leq\frac{(a+b+c)^2}{3} \quad \textbf{(1p)}$$

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SUBIECTUL 2.

Să se demonstreze egalitatea:

$$1+\sum_{k=0}^{n}\left(\frac{k+3}{k+1}\cdot C_{2k+3}^{k}\right)=C_{2n+3}^{n+1}+\sum_{k=0}^{n}\left(\frac{2k+1}{k+1}\cdot C_{2k}^{k}\right)$$

Soluție:

$$\sum_{k=0}^{n}\left(\frac{k+3}{k+1}\cdot C_{2k+3}^{k}\right)-\sum_{k=0}^{n}\left(\frac{2k+1}{k+1}\cdot C_{2k}^{k}\right)=$$

$$=\sum_{k=0}^{n}\left(\frac{k+3}{k+1}\cdot C_{2k+3}^{k}-\frac{2k+1}{k+1}\cdot C_{2k}^{k}\right)$$ **(1p)**

$$\sum_{k=0}^{n}\left(\frac{k+3}{k+1}\cdot C_{2k+3}^{k}-\frac{2k+1}{k+1}\cdot C_{2k}^{k}\right)=\sum_{k=0}^{n}\left(\frac{k+3}{k+1}\cdot\frac{(2k+3)!}{k!(k+3)!}-\frac{2k+1}{k+1}\cdot\frac{(2k)!}{k!k!}\right)$$ **(1p)**

$$\sum_{k=0}^{n}\left(\frac{k+3}{k+1}\cdot\frac{(2k+3)!}{k!(k+3)!}-\frac{2k+1}{k+1}\cdot\frac{(2k)!}{k!k!}\right)=\sum_{k=0}^{n}\left(\frac{(2k+3)!}{(k+1)!(k+2)!}-\frac{(2k+1)!}{(k+1)!k!}\right)$$ **(1p)**

$$\sum_{k=0}^{n}\left(\frac{(2k+3)!}{(k+1)!(k+2)!}-\frac{(2k+1)!}{(k+1)!k!}\right)=\sum_{k=0}^{n}\left(C_{2k+3}^{k+1}-C_{2k+1}^{k}\right)$$ **(2p)**

$$\sum_{k=0}^{n}\left(C_{2k+3}^{k+1}-C_{2k+1}^{k}\right)=C_{3}^{1}-C_{1}^{0}+C_{5}^{2}-C_{3}^{1}+C_{7}^{3}-C_{5}^{2}+\ldots\ldots\ldots+$$
$$+C_{2n+3}^{n+1}-C_{2n+1}^{n}$$ **(1p)**

$$C_{3}^{1}-C_{1}^{0}+C_{5}^{2}-C_{3}^{1}+C_{7}^{3}-C_{5}^{2}+\ldots\ldots+C_{2n+3}^{n+1}-C_{2n+1}^{n}=C_{2n+3}^{n+1}-C_{1}^{0}==C_{2n+3}^{n+1}-1$$ **(1p)**

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SUBIECTUL 3.

Fie $\varepsilon_k = \cos\frac{2k\pi}{n} + i\sin\frac{2k\pi}{n}$, $n \in \mathbf{N}^*$, $k \in \mathbf{N}$, $k < n$, $\varepsilon_n = \varepsilon_0$.

Arătați că $n \cdot \sin\frac{2\pi}{n} < \sum_{k=1}^{n} \left|\varepsilon_k - \varepsilon_{k-1}\right| < 2\pi$.

Soluție:

ε_k=afixele poligonului regulat înscris în cercul unitate, (**2**p)

de perimetru $\sum_{k=1}^{n} \left|\varepsilon_k - \varepsilon_{k-1}\right| < 2\pi$ = lungimea cercului; (**2**p)

$\left|\varepsilon_k - \varepsilon_{k-1}\right| = 2\sin\frac{\pi}{n}$, (**2**p)

finalizarea(**1**p)